

## Symmetry and the order of events in time: description of a reversible thermal energy converter composed of negative mass

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**Abstract:** In a previous paper [1], we analyzed the sequence of events that occurs in the time domain of a reversible cyclical thermal energy converter using the Gibbs equation and showed that even when there is no net change of entropy there is an asymmetrical order of quasi-static states along the time axis, when thermal energy is converted into mechanical pressure-volume work. The spontaneous evolution towards an equilibrium state, which is characterized by the unidirectional transport of entropy from the hot reservoir to the cold reservoir, is coincident with this asymmetrical order of events. In this paper, we “mend” the above mentioned asymmetry by writing the Gibbs equation for a class of substances that have negative mass. Such extraordinary substances would allow a reversible cyclical energy converter to proceed in the reverse order coincident with a unidirectional transport of entropy from the cold reservoir to the hot reservoir and a spontaneous evolution away from equilibrium. By taking into consideration thermodynamic systems composed of positive or negative mass, we have made the Second Law of Thermodynamics symmetrical in terms of entropy flow. The symmetrical Second Law of Thermodynamics states that, for reversible thermodynamic systems composed of positive mass, entropy flows from the hot reservoir to the cold reservoir and for reversible thermodynamic systems composed of negative mass, entropy flows from the cold reservoir to the hot reservoir. Consequently, systems of positive mass evolve towards equilibrium while systems of negative mass evolve away from equilibrium. This is reminiscent of Feynman’s definition of antimatter as matter going backward in time. In another paper in this series [2], we suggested that substances with negative mass are real and synonymous with antimatter.

**Key words:** Carnot cycle, Gibbs equation, negative mass, reversibility, Second Law of Thermodynamics, symmetry, time direction

### 1. Introduction

A defining strategy of Einstein’s approach to physics was to assume that the correct form of the Laws of Nature were formally symmetrical and that problems in physics could be identified and solved by uncovering asymmetries in fundamental theories and discovering a way to make them symmetrical [3, 4, 5]. The Second Law of Thermodynamics, unlike the other Laws of Nature, is asymmetrical with respect to the predicted order of events in time, and consequently the Second Law of Thermodynamics can be used to understand both natural and artificial processes that evolve asymmetrically in time, be they physical, chemical, biological, atmospheric, engineering, economic or societal [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. We have shown that the Carnot cycle, which served as the basis of the Second Law of Thermodynamics, when analyzed on the temperature-entropy plane, converts thermal energy to mechanical pressure-volume work only when it runs through a sequence of quasi-static processes in the clockwise direction, indicating that there is an

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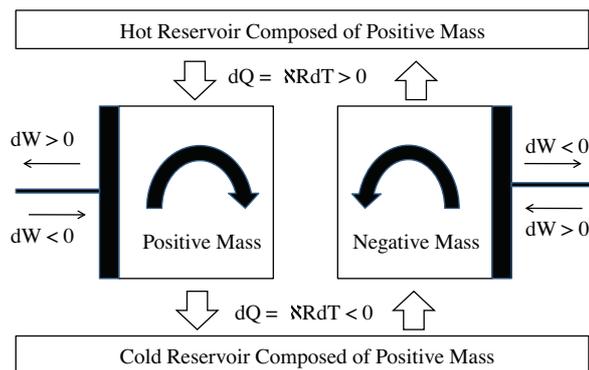
asymmetry in nature described by the thermodynamic equations even when there is no net change of entropy [29]. The quasi-static processes, through which the system passes when it acts as a thermal energy converter, can be considered to be a sequence of events in time. Consequently, in a thermal energy converter, even when there is no net change of entropy, there is an asymmetry in the order of events in time.

In the quest for symmetry in a physics that has evolved by reducing the “*different and the changing*” to the “*identical and the permanent*,” the common sense notion of time, with its unidirectional and asymmetrical nature, has been a causality [30, 31, 32]. One way of achieving symmetry has been to minimize the importance of the observations of asymmetry in the order of events in time and to deny the reality of time [33, 34, 35, 36, 37] by considering time to be merely a mathematical parameter in real equations that describe the illusion of time. Another way of achieving symmetry has been to consider time itself to flow bidirectionally [38, 39, 40, 41, 42]. However, the question arises: is it possible to achieve symmetry in the order of events in time by reconceptualizing physics and accepting the reality of time and its unidirectional, asymmetrical nature [30, 43], and then generalizing the Second Law of Thermodynamics?

*A priori*, in order to achieve symmetry in the order of events in time when one assumes that time itself flows asymmetrically, one must introduce into the thermodynamic equations the possibility that one of the parameters, heretofore considered to be only positive, can be either positive or negative for two complementary classes of substances. The “relative negative sign,” which represents the complementary substance, then transforms the original equations so that using the complementary substance, the Carnot cycle runs in the anticlockwise direction when it transforms thermal energy into mechanical pressure-volume work. Based on the principles of symmetry, we postulate that such a class of substances exists and propose that the Second Law of Thermodynamics, which dictates the order of events in time in reversible thermodynamic systems, can be made symmetrical. Figure 1 illustrates thermal energy converters composed of complementary substances that are powered by the same thermal gradient.

Here we propose a new discrete symmetry that assumes that ordinary matter has a positive mass and that an extraordinary type of matter exists that has a negative mass, and we show that the observed origin of the asymmetry in the order of events in time seen in the Second Law of Thermodynamics [29] results from the fact that only substances with positive mass were experimentally observed and theoretically considered. Substances with negative mass have been postulated to exist by others [44, 45, 46, 47, 48, 49, 50, 51, 52]. Negative mass is not a fictional characteristic, as we have previously shown that antimatter can be considered to have a negative mass [53] and thus antimatter may be the reification of the extraordinary matter introduced here to obtain symmetry in the Second Law of Thermodynamics. Whether or not the discrete mass symmetry [53] proposed here relates to a new conservation law [46] in the manner that Noether’s Theorem states that continuous symmetries in time and space relate to conservation laws regarding energy, linear momentum and angular momentum [54, 55, 56], is unknown.

We start with the assumption that the Second Law of Thermodynamics is a fundamental Law of Nature that describes the order of events in time, even in reversible processes, in a scale-invariant manner. That is, the Second Law of Thermodynamics makes no distinction between macroscopic and microscopic processes. However, there are two schools of thought when it comes to the scale-invariance of the Second Law of Thermodynamics [57, 58, 59]. One school asserts that the Second Law of Thermodynamics is a fundamental Law of Nature and the other school asserts that the Second Law of Thermodynamics is not a fundamental law at all, but an incomplete expression of statistical mechanics and “*a very strong statement about probabilities*” [60]. The utility of the Second Law of Thermodynamics in understanding microscopic processes is supported by the fact



**Figure 1.** Reversible cyclical thermal energy converters composed of either an ordinary substance with positive mass or an extraordinary substance with negative mass.  $dQ > 0$  when the thermal energy converters are in thermal communication with the hot reservoir, and  $dQ < 0$  when the thermal energy converters are in thermal communication with the cold reservoir. When a system composed of positive mass is in thermal communication with a hot reservoir, heat flows into the system. When a system composed of positive mass is in thermal communication with a cold reservoir, heat flows out of the system. When a system composed of negative mass is in thermal communication with a hot reservoir, heat flows out of the system. When a system composed of negative mass is in thermal communication with a cold reservoir, heat flows into the system. Consequently, in response to a thermal gradient, systems made of positive mass catalyze the spontaneous evolution towards equilibrium, while systems made of negative mass catalyze the spontaneous evolution away from equilibrium.  $dW > 0$  when the gas in the thermal energy converter composed of positive mass expands and when the gas in the thermal energy converter composed of negative mass contracts. When transforming thermal energy into mechanical pressure-volume work, the thermal energy converter made of positive mass proceeds clockwise through a series of quasi-static processes while the thermal energy converter made of negative mass proceeds anticlockwise through the same series of quasi-static processes. The transformations are accompanied by a transport of entropy from the hot reservoir to the cold reservoir in the case of a thermal energy converter composed of positive mass and from the cold reservoir to the hot reservoir in the case of a thermal energy converter composed of negative mass. While the tandem operation of a thermal energy converter composed of positive mass and a thermal energy converter composed of negative mass would seem to lead to a perpetual motion machine, it would not, since the operation of a thermal energy converter requires two different temperatures and, according to the optomechanical model [43, 82], at any temperature greater than absolute zero, friction exists.

that Planck gave birth to the quantum theory from the cradle of the Second Law of Thermodynamics [61]. Planck wrote, “*The two laws, it seems to me, must be upheld under all circumstances. For the rest, I was ready to sacrifice every one of my convictions about the physical laws*” [62]. Einstein [63] also believed that the laws of thermodynamics would hold for all physical processes and wrote, “*...classical thermodynamics... is the only physical theory of universal content concerning which I am convinced that, within the framework of the applicability of its basic concepts, it will never be overthrown.*” More recently, Pippard [64] stated that “*there is thus no justification for the view, often glibly repeated, that the Second Law of Thermodynamics is only statistically true, in the sense that microscopic violations repeatedly occur but never violations of any serious magnitude. On the contrary, no evidence has ever been presented that the second law breaks down under any circumstances, and even the entropy law appears to have almost universal validity, except in such futile experiments as we have discussed above, the removal and reapplication of constraints.*” Likewise, Longair [62] stated that “*...the two laws of thermodynamics have never been found to be wanting in any physical problem.*” We argue that the symmetrical Second Law of Thermodynamics, which makes use of positive and negative

mass, will determine the order of events in time of all macroscopic and microscopic processes that are subjected to an input or output of electromagnetic energy.

In order to explore the physical properties of extraordinary matter, we take a mechanical approach similar to that taken by Bernoulli [65], Herapath [66], Waterston [67], Joule [68], Clausius [69], Maxwell [70] and Boltzmann [71], who used kinetic theory and statistical mechanics to relate classical mechanics with classical thermodynamics. We use the same line of reasoning, but start with the assumption that substances could have either positive or negative mass [44, 45, 46, 47, 48, 49, 50, 51, 52]. This possibility had been virtually neglected because Newton [72] stated his Second Law only in terms of positive masses. This was a reasonable constraint since no other type of matter that could be interpreted to have a negative mass was known at that time. Newton stated his Second law of mechanics thusly, “*The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.*” In this paper we define a substance with positive mass as a substance that accelerates parallel to a causal force and a substance with negative mass as a substance that accelerates antiparallel to a causal force.\* Algebraically, positive mass and negative mass are defined by the relations

$$m^+ = \frac{\vec{\mathbf{F}}}{\vec{\mathbf{a}}} \quad (1)$$

$$m^- = \frac{\vec{\mathbf{F}}}{\vec{\mathbf{a}}}, \quad (2)$$

where  $\mathbf{F}$  denotes force,  $m$  denotes mass, which we propose can be positive or negative, and  $\mathbf{a}$  denotes the acceleration of the particle. The oppositely-directed movements of complementary substances to a given force or field have precedents in physics. For example, in a given electric field, a positive charge moves antiparallel to the movement of a negative charge.

In the Newtonian world, mass is defined as the ratio of the applied force to the acceleration of the mass, and the force is defined as the product of the mass and the acceleration, that is

$$\mathbf{F} = m\mathbf{a} \quad (3)$$

Thus, in the Newtonian view, the definition of mass depends on the *a priori* definition of mass [73] and we are led to Poincaré’s [74] conclusion that “*Masses are co-efficients which are found convenient to introduce into calculations.*”

We now take both the causal force and the responsive force into consideration. In the case of positive mass, the acceleration of the particles that make up a gas and the direction of the causal force it exerts will be parallel. Likewise the direction of the responsive force of the piston and the acceleration of the piston will be parallel. Consequently, a particle accelerating toward a piston will cause the piston to accelerate in a direction parallel to the casual force. Taking the force of the particle on the piston as well as the response of the piston to the particle into consideration, the piston will move away from the gas.

In the case of negative mass, by contrast, the sign of the force and the sign of the acceleration will be antiparallel. In the case of negative mass, the acceleration of the particles that make up the gas and the direction of the causal force it exerts will be antiparallel. Likewise the direction of the responsive force of the

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\*It is important to distinguish between a causal force and the combination of a causal force and a responsive force when one considers the action of negative mass on other negative masses. The accelerations of positive mass and negative mass particles to a given perturbation will always be in opposite directions.

piston composed of negative mass and its acceleration will also be antiparallel. Consequently, a negative mass particle accelerating toward a negative mass piston will cause the piston to accelerate in a direction antiparallel to the casual force. Taking the force of the particle on the piston as well as the response of the piston to the particle into consideration, the piston will move away from the gas. As we will show below, the piston will move away from a gas composed of positive mass in communication with a hot reservoir and a gas composed of negative mass in communication with a cold reservoir.

The above definition of mass allows for positive and negative kinetic energies. Positive mass particles have positive kinetic energies and negative mass particles have negative kinetic energies. Using the identity  $dx/dt = v$  and taking the integral with respect to  $x$ , we get

$$\int \mathbf{F}dx = m \int (dv/dt)dx = m \int (dx/dt)dv = m \int vdv = \frac{1}{2}mv^2 + C, \quad (4)$$

where  $C$  is a constant of integration,  $\frac{1}{2}mv^2$  is kinetic energy and  $\int \mathbf{F}dx$  denotes an energy input ( $E$ ) that can raise the kinetic energy at nonrelativistic velocities from 0 to  $\frac{1}{2}mv^2$  to an accuracy of the constant of integration. The kinetic energy of a particle is given by

$$E = \frac{1}{2}mv^2. \quad (5)$$

As we will show below, when putting a system with constant volume composed of positive mass in thermal communication with a hot reservoir, the kinetic energies and speeds of the particles will increase (i.e. become more positive)<sup>†</sup>. By contrast, when putting a system with constant volume composed of negative mass in thermal communication with a hot reservoir, the kinetic energies and speeds will decrease (i.e. kinetic energies become more positive and speeds become less positive). Radiative transfer, between a hot reservoir and various test systems at constant volume, accelerates particles in an ideal gas system composed of positive mass and decelerates particles in an ideal gas system composed of negative mass. We consider the sign of the acceleration resulting from the perturbation caused by thermal communication with a hot reservoir to be related to the order of events in time. Thus, when subjected to a given thermal perturbation, the order of events in the time domain is reversed for a system composed of positive mass compared to a system composed of negative mass.

While collisions of particles with a piston in a thermal energy converter composed of either positive mass or negative mass cause an outwardly directed pressure, as we will show below, under isothermal conditions, the exchange of thermal energy between a gas in a thermal energy converter and a hot reservoir causes a gas composed of positive mass to expand and a gas composed of negative mass to contract. Given the history of defining temperature by its ability to cause an expansion of a gas or a liquid [75, 76], it would be reasonable to define positive masses as having positive temperatures and negative masses as having negative temperatures. While the expansion of a substance upon being put in thermal communication with a hot body is generally—although not always—correlated with an increase in the temperature of the substance, here we have opted to use positive temperatures for both positive and negative masses in order to facilitate the comparison between thermal energy converters composed of positive mass or negative mass under identical experimental conditions. Since temperature, an intrinsic parameter, and entropy, an extrinsic parameter, are the conjugate factors that

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<sup>†</sup>We assume that all reservoirs are infinite in volume, composed of positive mass, and communicate thermally with the systems via radiative transfer (photon exchange). The flow of heat into or out of a thermal energy converter is denoted by  $dQ$ . The sign convention is defined so that  $dQ > 0$  when the thermal energy converter composed of either positive or negative mass is in thermal communication via radiative transfer with the hot reservoir and  $dQ < 0$  when the thermal energy converter composed of either positive or negative mass is in thermal communication via radiative transfer with the cold reservoir.

describe thermal energy [57], as a consequence of defining the temperatures of systems composed of positive mass and negative mass to be routinely positive, the direction of flow of entropy through a thermal energy converter composed of positive mass or negative mass systems will be by necessity, from the hot reservoir to the cold reservoir in systems composed of positive mass and from the cold reservoir to the hot reservoir in systems composed of negative mass. Consequently, in response to a thermal gradient, systems made of positive mass catalyze the spontaneous evolution towards equilibrium, while systems made of negative mass catalyze the spontaneous evolution away from equilibrium.

## 2. Microscopic behavior of an ideal gas composed of positive or negative mass

Consider a closed system composed of a constant cylindrical volume  $V$  of  $N$  particles of a monatomic gaseous substance with either a positive or negative mass  $m$  traveling at a mean square velocity of  $\langle v^2 \rangle$ . It is considered a homogeneous system in which the containers are made out of the same kind of substance as the enclosed gas (i.e. all positive or all negative mass). At a given temperature, the gas will exert a pressure  $P$  on a frictionless piston according to the following equation based on Newton's Second Law:

$$\aleph P = \frac{Nm \langle v^2 \rangle}{3V}. \quad (6)$$

When positive mass particles accelerate toward the piston, they impart a force that is parallel to their accelerations. The piston, which is also composed of positive mass, responds to this force by accelerating in a direction parallel to the force. Thus the piston moves as if it were exposed to positive pressure. When negative mass particles accelerate toward the piston, they impart a force that is antiparallel to their acceleration. This occurs because, with negative mass, the direction of the force vector is opposite to the direction of acceleration. The piston, which is composed of negative mass, responds to this force by accelerating antiparallel to it. Consequently, two negatives make a positive and the negative mass piston moves as if it were exposed to a positive pressure. Thus the pistons respond as if particles with positive or negative mass exert a positive pressure (i.e. a positive force per unit area) on their respective pistons.

Since the pressure is deemed positive in both kinds of homogeneous systems, yet the right side of equation (6) is positive for positive mass and negative for negative mass, we have to insert a variable into the equation that takes into consideration the sign of the mass of the piston. This variable is denoted by  $\aleph$ , and it is equal to  $+1$  for positive mass and  $-1$  for negative mass.  $\aleph$  represents the relation between the direction of the force and the direction of movement and will be defined more precisely below.

Using a partially modified form of the ideal gas law ( $\aleph PV = nRT$ ), we get the following unfinished equation:

$$\aleph PV = \frac{Nm \langle v^2 \rangle}{3} = nRT, \quad (7)$$

where  $n$  is the number of moles of the gaseous substance,  $R$  is the Universal Gas Constant and  $T$  is the absolute temperature.

The Universal Gas Constant  $R$  and consequently the value of Boltzmann's Constant are not absolute constants but *relative* constants that depend on the definition of the relative atomic mass standard (e.g., H = 1 amu; O = 16 amu;  $^{12}\text{C}$  = 12 amu)<sup>‡</sup>. If we keep  $R$  and  $k$  immutable, based on their value relative to

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<sup>‡</sup>  $R$  can be determined by measuring the pressure of one mole of gas in a defined volume at a given temperature using the

$^{12}\text{C}$ , then the value of  $\aleph$  is given by the following equation:

$$\aleph = \frac{12 \text{ amu}}{^{12}\text{C}} \cdot \frac{6.02 \times 10^{23} \text{ }^{12}\text{C}}{\text{mol}} \cdot \frac{\pm 1.66053886 \times 10^{-27} \text{ kg}}{\text{amu}} \cdot \frac{\text{mol}}{0.012 \text{ kg}} \quad (8)$$

According to our definition, the mass of the relative atomic mass unit (amu) of a substance made of positive mass is  $1.66053886 \times 10^{-27}$  kg, while the mass of the relative atomic mass unit of a substance made of negative mass is  $-1.66053886 \times 10^{-27}$  kg. Given these magnitudes,  $\aleph = +1$  for positive mass and  $\aleph = -1$  for negative mass. Use of the coefficient  $\aleph$  emphasizes that the magnitude of Boltzmann's Constant and the Universal Gas Constant depend upon the value of the atomic mass standard. Loeb [77] first introduced a coefficient to emphasize that the atomic masses he was discussing were based on the atomic mass of oxygen being 16.000 instead of the atomic mass of hydrogen being 1.

We assume that the Third Law of Thermodynamics [78] holds for both positive mass and negative mass, that all temperatures of thermodynamic systems are positive, that the temperature of the hot reservoir ( $T_H$ ) is always more positive than the temperature of the cold reservoir ( $T_C$ ), and that the temperature of the gas in the thermal energy converter varies cyclically between  $T_H$  and  $T_C$ . However, with these assumptions,  $R$  would be positive when  $m$  is positive and negative when  $m$  is negative. In order to avoid confusion in the use of these constants, we maintain that the Universal Gas Constant is always positive and equal to  $8.31 \text{ J mol}^{-1} \text{ K}^{-1}$  and Boltzmann's Constant  $k$ , which is the ratio of the Universal Gas Constant to Avogadro's Number, is always positive and equal to  $1.38 \times 10^{-23} \text{ J/K}$ . Consequently, we also have to introduce the dimensionless coefficient  $\aleph$  to account for the sign of the mass in equations that relate the mass to temperature.  $\aleph$  is positive when the mass is positive and negative when the mass is negative:

$$\aleph PV = \frac{Nm \langle v^2 \rangle}{3} = n\aleph RT. \quad (9)$$

Consequently, the fully-modified equation of state for an ideal gas becomes  $\aleph PV = n\aleph RT$ . Using the facts that  $n \equiv \frac{N}{N_A}$  and  $R/N_A \equiv k$ , we get the relationship between the kinetic energy per particle of the monatomic gas and its temperature:

$$\frac{1}{2}m \langle v^2 \rangle = \frac{3}{2}\aleph kT. \quad (10)$$

The left side of the above equation represents the average kinetic energy per particle of the gas in question. Thus when  $m$  is positive, the kinetic energy of the gas is positive and when  $m$  is negative, the kinetic energy of the gas is negative. As the temperature of a gas made of positive mass increases, the kinetic energy of the gas and consequently, at non-relativistic speeds, its mean square velocity gets proportionally more positive. When the mass of a gas is negative, the kinetic energy of the gas is negative. As the temperature of a gas made of negative mass increases, at non-relativistic speeds, the kinetic energy increases (i.e. gets more negative) and consequently the mean square velocity increases (i.e. gets more positive). Thus in both cases, the mean square velocity is proportional to the temperature of the gas. As we will show below, the temperature of a gas made of positive mass in thermal communication with a hot reservoir increases, while the temperature of a gas made of negative mass in thermal communication with a hot reservoir decreases.

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relation  $R = \frac{PV}{T}$ . Historically, as the atomic mass standard changed from H = 1 to O = 16 to  $^{12}\text{C} = 12$ , the relative atomic mass of hydrogen changed from 1 amu to 1.0080 amu to 1.007277 amu. Thus the pressure produced by one mole of hydrogen, the mass of which has gone from 1 g to 1.0080 g to 1.007277 g increases, and the value of  $R$  calculated from it also increases. Thus we see that the value of the universal gas constant depends on the atomic mass standard used to determine it.

After multiplying both sides of equation 10 by Avogadro's Number, we get the relation between the total kinetic energy of a mole of a monatomic gas and the temperature:

$$U = \frac{1}{2}N_A m \langle v^2 \rangle = \frac{3}{2}\aleph RT. \quad (11)$$

We define the internal energy per mole ( $U$ ) of an ideal monatomic gas at constant volume as  $\frac{3}{2}\aleph RT$ , which is equal to the total kinetic energy per mole ( $\frac{1}{2}N_A m \langle v^2 \rangle$ ). Like kinetic energy,  $U$  is positive when the mass is positive and negative when the mass is negative. Since the mass symmetry is discrete, the least energy state is when  $U = 0$  for both positive mass and negative mass systems. The change in the internal energy of a system with constant volume in response to being in communication with a hot reservoir or a cold reservoir is given by the following equation:

$$dU = \frac{3}{2}\aleph R dT = dQ. \quad (12)$$

The flow of heat into or out of a thermal energy converter is denoted by  $dQ$ , where  $dQ > 0$  when the thermal energy converter is in thermal communication with the hot reservoir, and  $dQ < 0$  when the thermal energy converter is in thermal communication with the cold reservoir. In the case of a system composed of positive mass, when  $dQ > 0$ , heat flows into the system from the hot reservoir and the internal energy of the system becomes more positive ( $dU > 0$ ). By contrast, in the case of a system composed of negative mass, when  $dQ > 0$ , heat flows from the system to the hot reservoir and the internal energy of the system becomes less negative ( $dU > 0$ ). When in thermal communication with a cold reservoir, heat ( $dQ < 0$ ) flows out of a system composed of positive mass, and the internal energy of the system becomes less positive ( $dU < 0$ ). By contrast, when a system composed of negative mass is in thermal communication with a cold reservoir, heat ( $dQ < 0$ ) flows into the system and the internal energy of the system becomes more negative ( $dU < 0$ ).

After rearranging equation 12, we get the following relationship that describes what happens to the temperature of a system with a constant volume when it is in thermal communication with the radiation of a reservoir, hotter ( $dQ > 0$ ) or colder ( $dQ < 0$ ) than the system:

$$dT = \frac{2}{3}dQ/\aleph R \quad (13)$$

According to equation 13, putting a system in thermal communication with the radiation of a hot reservoir will cause the temperature of a system composed of positive mass to rise to the level of  $T_H$  and the temperature of a system composed of negative mass to fall below the level of  $T_H$ , perhaps to  $T_C$ . In order to raise the temperature of a gas made of negative mass to  $T_H$ , one would have to put the gas in thermal communication with the radiation of a cold reservoir. Thus if one were to define the direction of heat flow in terms of its ability to raise the temperature of a gas, one would say that in order to raise the temperature of a gas made of positive or negative mass, heat should flow from the hot reservoir to the gas made of positive mass and from the cold reservoir to the gas made of negative mass.

### 3. The First Law of Thermodynamics for systems composed of positive or negative mass

Based on the sign conventions assumed in the mechanical approach taken above, we will interpret the First Law of Thermodynamics [69] for a closed system composed of an ideal monatomic gas made of positive or negative mass and use it to define the specific heats of substances composed of positive or negative mass. We define work

as being positive when a gas composed of positive mass expands and a gas composed of negative mass contracts. Consider a system that can only expand or contract and the work done on or by the system is restricted to pressure-volume work ( $\aleph PdV$ ). Then

$$dU = dQ - \aleph PdV. \quad (14)$$

A gas composed of positive mass will approach the state of least internal energy ( $U = 0$ ) by putting the gas in thermal communication with a cold reservoir and/or increasing the volume of the gas. A gas composed of negative mass will approach the state of least internal energy ( $U = 0$ ) by putting the gas in thermal communication with a hot reservoir and/or increasing the volume of the gas. The state of an ideal gas given by its internal energy ( $U$ ) is completely determined by any two of the variables,  $P$ ,  $V$  or  $T$ . Let's use  $T$  and  $V$  to simplify the equations by reducing the number of variables. Since  $dU$  is a perfect differential,

$$dU = (\partial U/\partial T)_V dT + (\partial U/\partial V)_T dV \quad (15)$$

and equation 14 becomes

$$dQ = (\partial U/\partial T)_V dT + (\partial U/\partial V)_T dV + (\aleph P)_T dV. \quad (16)$$

$(\partial U/\partial T)_V$  is defined as the specific heat per mole at constant volume and is denoted by  $C_V$ . The internal energy of a substance with positive mass gets more positive when its temperature increases while the internal energy of a substance with negative mass gets more negative when its temperature increases. Thus  $C_V$  is positive for a positive mass and negative for a negative mass. Substituting  $C_V$  for  $(\partial U/\partial T)_V$  in equation 16 and assuming that the gas made of positive mass or negative mass is ideal,  $(\partial U/\partial V)_T$  vanishes and we get the relation

$$dQ = C_V dT + \aleph PdV. \quad (17)$$

Equation 17 contains the quantities  $P$ ,  $V$  and  $T$ . However, the fully modified equation of state ( $\aleph PV = \aleph RT$ ) allows us to eliminate any one of the variables in an ideal gas in order to express the First Law of Thermodynamics in terms of only two variables for any substance with a specific heat of  $C_V$ . In order to obtain an equation in terms of  $T$  and  $V$ , we eliminate  $P$  by substituting  $P = RT/V$ :

$$dQ = C_V dT + \left(\frac{\aleph RT}{V}\right) dV \quad (18)$$

Next, we obtain an equation in terms of  $P$  and  $T$  by eliminating  $dV$  using the relation  $V = RT/P$ . After differentiating  $V = (RT)P^{-1}$ , we get

$$dV = \left(\frac{R}{P}\right) dT - \left(\frac{RT}{P^2}\right) dP. \quad (19)$$

Substituting equation 19 into equation 18 and using  $\frac{PV}{RT} = 1$ , we get

$$dQ = (C_V + \aleph R)dT - \left(\frac{\aleph RT}{P}\right) dP \quad (20)$$

At constant pressure equation 20 becomes

$$dQ = (C_V + \aleph R)dT. \quad (21)$$

$(C_V + \aleph R)dT$  represents the quantity of heat which must be transferred between the system at constant pressure and a thermal reservoir in order to change the temperature of a mole of substance by  $dT$ . At constant pressure,  $\aleph R$  is the portion of the specific heat per mole that corresponds to the work of expansion ( $\aleph = +1$ ) or contraction ( $\aleph = -1$ ).  $C_V + \aleph R$  is denoted by  $C_P$ , the specific heat at constant pressure.  $C_P$  is positive for a system with positive mass and negative for a system with negative mass. The thermodynamic equations can be written in terms of  $C_P$ . For example, at constant pressure equation 18 becomes

$$dQ = (C_P - \aleph R)dT + \left(\frac{\aleph RT}{V}\right) dV. \quad (22)$$

The First Law of Thermodynamics for substances composed of positive or negative mass can be rewritten in terms of the two specific heats by substituting  $C_P - C_V$  for  $\aleph R$ :

$$dQ = (C_V)dT + \left(\frac{(C_P - C_V)T}{V}\right) dV. \quad (23)$$

According to the equipartition theorem, at constant volume,  $(\partial U/\partial T)_V = \frac{1}{2}\aleph R$  per degree of freedom. Thus the specific heat of a mole of an ideal monatomic gas composed of particles of positive or negative mass translating in three dimensions will be  $\frac{3}{2}\aleph R$ . The specific heat at constant volume of a mole of a diatomic gas composed of particles of positive or negative mass translating in three dimensions and rotating with two degrees of freedom will be  $\frac{5}{2}\aleph R$ . The specific heat at constant volume of a mole of a diatomic gas composed of particles of positive or negative mass translating in three dimensions, rotating with two degrees of freedom, and vibrating with two degrees of freedom will be  $\frac{7}{2}\aleph R$ .

#### 4. The Second Law of Thermodynamics for a reversible cyclical system composed of positive or negative mass

The Second Law of Thermodynamics was originally developed to explain the direction of heat flow in a reversible cyclical thermal energy converter composed of a fixed amount of an ideal gas with positive mass that was restricted to do mechanical pressure-volume work [69]. The Second Law of Thermodynamics was not developed to account for systems composed of negative mass. In order to develop the Second Law of Thermodynamics for substances composed of positive or negative mass, we must first determine the relationship between a change in temperature and a change in volume of a gas during an adiabatic process. For an adiabatic process where  $dQ = 0$ , equation 23 becomes

$$0 = (C_V)dT + \left(\frac{(C_P - C_V)T}{V}\right) dV. \quad (24)$$

Divide both sides by  $TC_V$  and integrate:

$$\ln T + \left(\frac{C_P}{C_V} - 1\right) \ln V = \text{constant of integration.} \quad (25)$$

We define  $\gamma$  to be the ratio of the two specific heats ( $C_P/C_V$ ). Since  $C_P$  and  $C_V$  are both positive for positive mass and negative for negative mass,  $\gamma$  is always positive for substances made of positive mass or negative mass. After eliminating the logarithms by exponentiating and after substituting  $\gamma$  for  $\frac{C_P}{C_V}$ , we get

$$TV^{(\gamma-1)} = \text{constant of integration.} \quad (26)$$

We can eliminate the constant of integration by allowing  $T_o$  and  $V_o$  to represent the initial values of  $T$  and  $V$ , respectively:

$$\frac{T}{T_o} = \left(\frac{V_o}{V}\right)^{(\gamma-1)}. \quad (27)$$

Under adiabatic conditions, where there is no exchange of thermal or any other kind of electromagnetic energy,  $T$  decreases exponentially with  $V$  for a gas made of positive mass or negative mass. Upon expansion, the temperature of the gas in the reversible thermal energy converter composed of positive or negative mass will fall from  $T_H$  to  $T_C$ , and upon contraction, the temperature of the gas in the reversible thermal energy converter composed of positive or negative mass will rise from  $T_C$  to  $T_H$ .

Carnot [79] discovered that in a thermal energy converter some heat was converted to mechanical pressure-volume work while the rest was transferred from the hot reservoir to the cold reservoir and that the ratio of the heat that could perform work to the heat that was transferred to the cold reservoir was independent of the nature of the substance. Clausius [69] discovered a function ( $\phi$ ) that related the amount of heat ( $dQ$ ) transferred from a hot reservoir to a cold reservoir through a thermal energy converter to its temperature ( $T$ ) and he called that function, entropy. As we showed above, when a substance with positive mass is in thermal communication with a hot reservoir, the temperature of the substance rises until it reaches the temperature of the hot reservoir, but when a substance with negative mass is in thermal communication with a hot reservoir, the temperature of that substance falls. Thus, the sign of the function that relates heat flow to temperature depends on whether the substance has a positive mass or a negative mass. Consequently, in order to take into consideration substances with either positive mass or negative mass, we include the coefficient  $\aleph$  and write the function like so:

$$dQ = \aleph\phi(T_H, T_C). \quad (28)$$

In light of the formal possibility of positive and negative mass, it is interesting to read a quote from Planck [57] in which he discusses the entropy of a single particle in terms of its mass, specific volume and temperature: *“It must not be supposed that the entropy of a gas has meaning only for states of equilibrium. We may assume each sufficiently small particle, even of a gas in turmoil, to be homogeneous and at a definite temperature, and must . . . assign to it a definite value of the entropy.  $M$ ,  $v$ , and  $T$  are then the mass, the specific volume, and temperature of the particle under consideration. A summation extending over all the particles of the mass—within which the values of  $v$  and  $T$  may vary from particle to particle—gives the entropy of the whole mass of the gas in the particular state.”*

To find the function that describes entropy, we will model the reversible cyclical thermal energy converter containing an ideal gas undergoing quasi-static processes where there are two isothermal processes and two adiabatic processes (Figure 2). Under isothermal conditions, in which  $dU = 0$ , the First Law of Thermodynamics in the form of Equation 18 can be written

$$dQ = (\aleph RT) \frac{dV}{V}, \quad (29)$$

where  $T$  is the temperature of the reservoir in thermal communication with the gas. Integrating equation 29, under isothermal conditions, where a system composed of positive mass or negative mass is connected to a hot ( $T_H$ ) reservoir so that  $Q_H > 0$ , we get

$$Q_H = \aleph RT_H \ln \frac{V_2}{V_1}. \quad (30)$$

When in thermal communication with a hot ( $T_H$ ) reservoir, a positive mass expands and a negative mass contracts. Integrating equation 29, under isothermal conditions, where a system composed of positive mass or negative mass is connected to a cold ( $T_C$ ) reservoir so that  $Q_C < 0$ , we get

$$Q_C = \aleph RT_C \ln \frac{V_4}{V_3}. \quad (31)$$

When in thermal communication with a cold ( $T_C$ ) reservoir, a positive mass contracts and a negative mass expands. Combine equations 30 and 31 to determine the net heat exchange divided by  $\aleph T$ , for a cyclical reversible thermal energy converter:

$$\frac{Q_H}{\aleph T_H} + \frac{Q_C}{\aleph T_C} = R \ln \frac{V_2}{V_1} + R \ln \frac{V_4}{V_3}. \quad (32)$$

Since  $\ln \frac{V_4}{V_3} = -\ln \frac{V_3}{V_4}$ , we get

$$\frac{Q_H}{\aleph T_H} + \frac{Q_C}{\aleph T_C} = R \left[ \ln \frac{V_2}{V_1} - \ln \frac{V_3}{V_4} \right]. \quad (33)$$

Below we show that the term  $[\ln \frac{V_2}{V_1} - \ln \frac{V_3}{V_4}]$  is equal to zero for substances with positive or negative mass. Consider one adiabatic process in the Carnot cycle:

$$\frac{T_H}{T_C} = \left( \frac{V_3}{V_2} \right)^{(\gamma-1)}. \quad (34)$$

For the other adiabatic process,

$$\frac{T_H}{T_C} = \left( \frac{V_4}{V_1} \right)^{(\gamma-1)}. \quad (35)$$

Thus  $\frac{V_4}{V_1} = \frac{V_3}{V_2}$ , and after rearranging,  $\frac{V_2}{V_1} = \frac{V_3}{V_4}$  and consequently, for a reversible cycle, equation 33 becomes

$$\frac{Q_H}{\aleph T_H} + \frac{Q_C}{\aleph T_C} = 0. \quad (36)$$

We define  $\frac{dQ}{\aleph T}$  as the change of entropy ( $dS$ ) in the thermal energy converter, composed of either positive mass or negative mass, operating at temperature  $T$  between  $T_H$  and  $T_C$ . That is, for positive or negative mass,

$$dS \equiv \frac{dQ}{\aleph T}. \quad (37)$$

Thus, the sign of the state function, known as entropy, which relates heat flow to temperature depends on whether the substance has a positive mass or a negative mass. This result follows from the equations presented above, in which we showed that when a substance with positive mass is in thermal communication with a hot reservoir, the temperature of the substance rises, but when a substance with negative mass is in thermal communication with a hot reservoir, the temperature of that substance falls.

Replacing  $dQ$  in equation 14 with  $\aleph T dS$  we get the fundamental law of thermodynamics for substances with positive or negative mass when all extensive parameters besides  $U$ ,  $S$  and  $V$  are constant<sup>§</sup>

$$dU = \aleph T dS - \aleph P dV. \quad (38)$$

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<sup>§</sup>The general form of this equation, known as the Gibbs equation, would be  $dU = \aleph \sum Y_i dX_i$ , where  $dX_i$  is a change in any extensive parameter and  $Y_i$  in the conjugate intensive parameter. The product  $Y_i dX_i$  gives the energy of type  $i$ .

The thermodynamic temperature is defined as  $(\partial U/\aleph\partial S)_V$  for positive and negative mass. Consistent with our treatment of temperature as generally having positive values,  $(\partial U/\aleph\partial S)_V$  is typically positive for both positive mass or negative mass although there are special cases where it can be negative [60, 80] for both.

Using the fundamental equation of thermodynamics for positive or negative mass, let us look at the operation of a reversible cyclical thermal energy converter [29] on the  $T$ - $S$  and the  $P$ - $V$  planes for substances composed of positive or negative mass:

$$\oint dU = \oint \aleph T dS - \oint \aleph P dV. \quad (39)$$

In the case of a reversible cycle,  $\oint dU = 0$  and

$$\oint \aleph T dS = \oint \aleph P dV. \quad (40)$$

That is, the magnitude of the thermal energy exchanged with the reservoirs ( $\oint \aleph T dS$ ) is equal to the magnitude of pressure-volume work ( $\oint \aleph P dV$ ).

We define the work ( $dW$ ) done by an ideal gas composed of positive or negative mass as

$$dW \equiv \aleph \oint P dV, \quad (41)$$

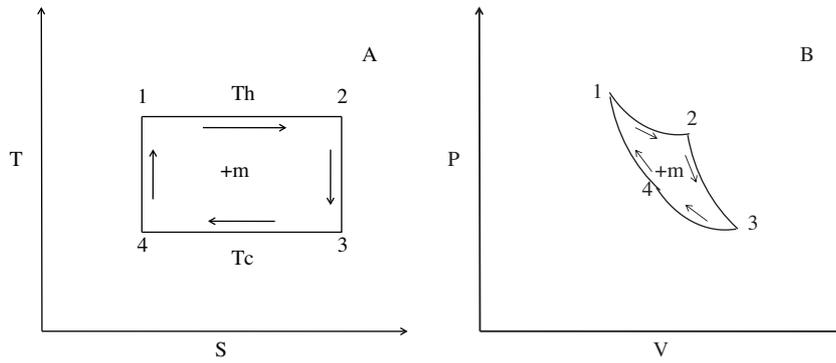
where the work done by the gas is defined by its response to being put in thermal communication with a hot ( $T_H$ ) reservoir. Work is positive when a gas composed of positive mass expands and a gas composed of negative mass contracts. By analogy, work by a positive mass gas acts to stretch a spring from its equilibrium position while work done by a negative mass gas acts to compress a spring from its equilibrium position. Equation 38 becomes

$$dU = \aleph T dS - dW. \quad (42)$$

Under isothermal conditions, the gas in a system composed of positive mass expands when the system is in communication with the radiation of a hot reservoir. During this process, thermal energy is converted into expansion work. By contrast, when a system composed of negative mass is in communication with the radiation of a hot reservoir, the exchange of thermal energy results in compression work.

When placed in a gradient between two thermal reservoirs,  $\aleph \oint T dS$  is positive for a system composed of positive mass since both  $\aleph$  and  $\oint T dS$  will be positive. It is also positive for a system composed of negative mass since both  $\aleph$  and  $\oint T dS$  will be negative. In systems composed of positive mass or negative mass, mechanical pressure-volume work can only be obtained reversibly from the thermal gradient when when  $\aleph \oint T dS$  and  $\aleph \oint P dV$  are both equal and positive. While thermal energy flows from the hot reservoir to the cold reservoir in the transformation of thermal energy into pressure-volume work in a reversible thermal energy converter made of positive mass, thermal energy flows from the cold reservoir to the hot reservoir during the transformation of thermal energy into pressure volume work in a reversible thermal energy converter made of negative mass.

The integration of  $\aleph \oint T dS$  is straightforward and simple for the conditions of the operation of a Carnot cycle (two isotherms and two adiabatics, where  $S_1 = S_4, S_2 = S_3, S_2 > S_1, S_3 > S_4$ ). The two adiabatic integrals vanish, as they are held at isentropic conditions ( $dS = 0$ ). Therefore the integration of the thermal



**Figure 2.** A reversible cyclical thermal energy converter composed of positive mass displayed on the temperature-entropy plane (A) and the pressure-volume plane (B) can be visualized like so: Step 1 ( $1 \rightarrow 2$ ): Microscopically, the positive mass particles exert a force on the piston that results in an expansion. The movement away from the gas will exert a Doppler-like force on the particles, causing them to move more slowly. Without any input of energy, the kinetic energy of the particles would decrease as would the temperature of the gas. Since thermal energy is exchanged with the hot ( $T_H$ ) reservoir, the gas remains isothermal and the velocities and kinetic energies of the particles remain constant. Macroscopically, when the cyclical thermal energy converter composed of positive mass is subjected to the hot reservoir, the gas in the thermal energy converter expands isothermally. During this process, a flux of thermal energy and entropy into the machine occurs which is coupled to the reversible pressure-volume work of expansion. Step 2 ( $2 \rightarrow 3$ ): Microscopically, as the piston moves away from the gas as a result of the collisions, the velocities of the particles decrease. Consequently, there is a decrease in the kinetic energy of the particles and the temperature of the gas. In contrast to the isothermal situation, under adiabatic conditions, there is no flow of thermal energy into or out of the system, so the kinetic energy and temperature continues to decrease to an arbitrary value. Macroscopically, when the thermal energy converter is no longer subjected to the hot reservoir, and it is in contact only with the reversible work reservoir, the gas expands adiabatically until its temperature falls to that of the cold reservoir. Step 3 ( $3 \rightarrow 4$ ): Microscopically, as the piston moves toward the gas, it provides a Doppler-like force on the particles, causing an increase in their velocity, their kinetic energy and the temperature of the gas. However, thermal energy is removed by the cold ( $T_C$ ) reservoir so that the velocities and kinetic energies of the particles, as well as the temperature of the gas, remain constant. Macroscopically, when the cyclical converter is subjected to the cold reservoir, there is an isothermal flux of thermal energy and entropy into the cold reservoir as the gas is compressed. This compression continues isothermally until the entropy reaches its initial value. Step 4 ( $4 \rightarrow 1$ ): Microscopically, as the piston moves toward the gas, it exerts a Doppler-like force on the particles, causing their velocities to increase. Consequently, there is an increase in the kinetic energies of the particles and the temperature of the gas. In contrast to the isothermal situation, under adiabatic conditions, there is no flow of thermal energy out of the system, so the kinetic energy and temperature continue to increase to an arbitrary value. Macroscopically, when the cold reservoir is no longer in communication with the gas, the adiabatic compression continues until the whole system returns to the initial condition. The net result of the cycle is the movement of entropy from the hot reservoir to the cold reservoir. This occurs without any change of the total entropy but with the transformation of thermal energy into mechanical pressure-volume work. During the reversible cycle, there is a net conversion of thermal energy into pressure-volume work because the compression at the lower temperature requires less pressure-volume energy than is produced during the expansion phase that occurs at the higher temperature. The physical properties and energetic relations of positive mass cause the substances in the thermal energy converter to behave in the way described above. That is, communication with a hot reservoir causes positive mass to expand and communication with a cold reservoir causes positive mass to contract. This behavior results in the clockwise ( $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$ ) operation of the cycle in the  $T$ - $S$  plane when producing mechanical pressure-volume work from thermal energy.

energy term in equation 38 is given by the equation

$$\aleph \oint TdS = \aleph \left[ T_H dS \int_1^2 + T_C \int_3^4 dS \right], \quad (43)$$

where  $T_H$  represents the temperature of the hot reservoir and  $T_C$  represents the temperature of the cold reservoir. Mathematically, the cycle can be operated in two directions, clockwise and anticlockwise. Integrating clockwise we find

$$\aleph \oint TdS = \aleph(T_H - T_C)(S_2 - S_1). \quad (44)$$

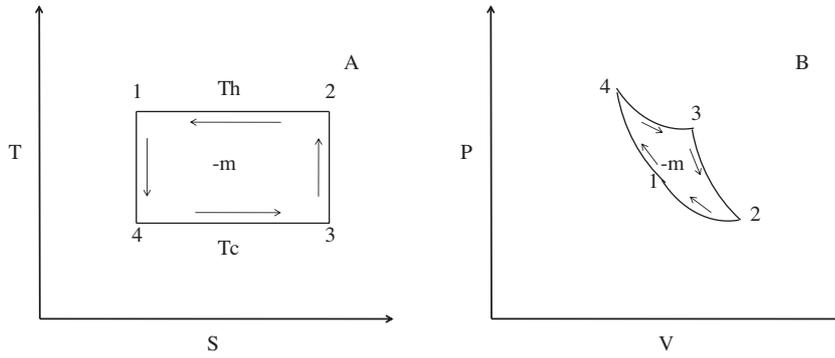
In the case of positive mass, when  $\aleph = +1$ ,  $(T_H - T_C) > 0$  and  $(S_2 - S_1) > 0$ ,  $\aleph \oint TdS$  is positive. Thus thermal energy can be converted into mechanical pressure-volume work when a thermal energy converter composed of positive mass operates clockwise. However, integrating clockwise in the case of negative mass, we find  $\aleph = -1$ ,  $(T_H - T_C) > 0$  and  $(S_2 - S_1) > 0$ , and thus  $\aleph \oint TdS$  is negative, indicating that thermal energy cannot be converted into mechanical pressure-volume work when a thermal energy converter composed of negative mass operates clockwise. By contrast, when we integrate anticlockwise we find

$$\aleph \oint TdS = \aleph(T_C - T_H)(S_2 - S_1). \quad (45)$$

In the case of positive mass, when  $\aleph = +1$ ,  $(T_C - T_H) > 0$  and  $(S_2 - S_1) > 0$ , then  $\aleph \oint TdS$  is negative and thermal energy cannot be converted into mechanical pressure-volume work when a thermal energy converter made of positive mass operates in the anticlockwise direction. However, integrating anticlockwise in the case of negative mass, we find that  $\aleph = -1$ ,  $(T_C - T_H) < 0$  and  $(S_2 - S_1) > 0$ , and consequently,  $\aleph \oint TdS$  is positive and thermal energy can be converted into mechanical pressure-volume work when a thermal energy converter made of negative mass operates in the anticlockwise direction.

The Carnot cycle can use positive mass to convert thermal energy to mechanical pressure-volume work only when it is operating in a clockwise direction. This means that the cycle only produces mechanical pressure-volume work when it operates in the following order of quasi-static processes:  $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$  along the positive direction of the time axis (Figure 2), and it does not produce mechanical work when it operates in the reverse order:  $4 \rightarrow 3 \rightarrow 2 \rightarrow 1 \rightarrow 4$ . By contrast, when substances with negative mass are used in the thermal energy converter, thermal energy is only converted into mechanical pressure-volume work when it is operating in the anticlockwise direction. This means that the cycle only produces mechanical pressure-volume work when it operates in the following order of quasi-static processes:  $4 \rightarrow 3 \rightarrow 2 \rightarrow 1 \rightarrow 4$  along the positive direction of the time axis (Figure 3), and it does not produce mechanical work when it operates in the reverse order:  $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1$ . The order of events in time for a thermal energy converter composed of positive mass is opposite to the order of events in time for a thermal energy converter composed of negative mass. Stated another way, a thermal energy converter composed of negative mass behaves like a thermal energy converter composed of positive mass, going backwards in time.

During the clockwise conversion of thermal energy into pressure-volume work in the reversible cyclical thermal energy converter composed of positive mass, entropy enters the system during the period of the high temperature isotherm and leaves the system during the period of the low temperature isotherm. By contrast, during the anticlockwise conversion of thermal energy into pressure-volume work in the reversible cyclical thermal energy converter composed of negative mass, entropy enters the system during the period of the low



**Figure 3.** pt A reversible cyclical thermal energy converter composed of negative mass displayed on the temperature-entropy plane (A) and the pressure-volume plane (B) can be visualized like so: Step 1 ( $4 \rightarrow 3$ ): Microscopically, as the negative mass particles collide with the piston, the piston moves away from the gas, resulting in expansion. The movement of the piston away from the gas will exert a Doppler-like force that will cause the gas particles to move more slowly. Without an influx of thermal energy and entropy from the cold ( $T_c$ ) reservoir, their kinetic energies and internal energy will become less negative (more positive). The temperature of the gas will decrease as the internal energy becomes more positive. Because of the negative specific heat of negative mass gases, thermal energy must be added by the cold reservoir to keep the velocities and kinetic energies of the particles, as well as the temperature of the gas, constant. Macroscopically, when the thermal energy converter composed of negative mass is in communication with the cold reservoir, there will be an isothermal flux of thermal energy and entropy from the cold reservoir to the gas, and the gas will expand isothermally. Step 2 ( $3 \rightarrow 2$ ): Microscopically, as the piston moves away from the gas, it exerts a Doppler-like force and the velocities of the particles decrease. Consequently, the kinetic energy of the particles becomes less negative and the temperature of the gas drops. In contrast to the isothermal situation, under adiabatic conditions, there is no flow of thermal energy into the system, so the kinetic energy and temperature continue to decrease to an arbitrary value. Macroscopically, when the thermal energy converter is no longer in communication with the cold reservoir, the temperature of the gas will decrease as it expands. Step 3 ( $2 \rightarrow 1$ ): Microscopically, as the piston moves toward the gas, it exerts a Doppler-like force causing the velocities of the particles to increase, the kinetic energy to get more negative, and the temperature of the gas to increase. In order to keep the kinetic energies of the particles as well as the temperature of the gas constant, the temperature must be prevented from increasing. Since the gas has a negative mass and a negative specific heat, it must be in thermal communication with the hot reservoir to keep the temperature of the gas from increasing. Macroscopically, when the cyclical converter is subjected to the hot reservoir, there is an isothermal flux of thermal energy and entropy from the gas to the hot reservoir, and the gas contracts. Step 4 ( $1 \rightarrow 4$ ): Microscopically, the piston moving toward the gas exerts a Doppler-like force that increases the velocities of the particles. Consequently, the kinetic energy of the gas becomes more negative. Under adiabatic conditions, in which there is no efflux of thermal energy, the temperature of the gas continues to increase to an arbitrary value as the gas contracts. Macroscopically, when the hot reservoir is no longer in communication with the gas, an adiabatic contraction occurs as the temperature of the system rises. The quasi-static states proceed in the following sequence:  $4 \rightarrow 3 \rightarrow 2 \rightarrow 1 \rightarrow 4$ . During the reversible cycle, there is a net conversion of thermal energy into pressure-volume work because the compression at the higher temperature requires less pressure-volume energy than is produced during the expansion phase that occurs at the lower temperature. The physical properties and energetic relations of negative mass cause the substances in the thermal energy converter to proceed in the anticlockwise ( $4 \rightarrow 3 \rightarrow 2 \rightarrow 1 \rightarrow 4$ ) direction in the  $T$ - $S$  plane when producing mechanical pressure-volume work from thermal energy.

temperature isotherm and leaves the system during the period of the high temperature isotherm. While in both cases, an increase in the entropy of the system is correlated with an expansion and a decrease in the entropy of the system is correlated with a contraction, the direction of entropy transport is not the same.

Entropy is transported from the hot reservoir to the cold reservoir during the transformation of thermal energy to mechanical pressure-volume work by a reversible energy converter composed of positive mass and from the cold reservoir to the hot reservoir during the transformation of thermal energy to mechanical pressure-volume work by a reversible energy converter composed of negative mass. We note that the analysis we have done in this paper depends on the assumptions that the thermal energy converter is reversible and that the reservoirs that provided the thermal gradient are composed of positive mass and infinite. In practice, the magnitude of the entropy flow discovered in the ideal reversible systems can be interpreted to be a measure of the dissipation or exacerbation of the thermal gradient used to drive the thermal energy converters that would occur in real systems composed of positive mass and negative mass, respectively, as a result of the fact that the heat reservoirs are not infinite.

The symmetrical Second Law of Thermodynamics states that, for a reversible system such as a reversible thermal energy converter composed of positive mass, entropy flows from the hot reservoir to the cold reservoir and for a reversible system, such as a reversible thermal energy converter composed of negative mass, entropy flows from the cold reservoir to the hot reservoir. Put another way, in a thermal energy converter, the exchange of electromagnetic radiation in the infrared region of the spectrum is correlated with the flow of entropy in opposite directions for systems composed of positive and negative mass. The formal possibility of the bidirectional flow of entropy is concordant with Einstein's retention of the advanced as well as the retarded potential in describing radiation processes [4, 5] as well as the retention of both the advanced and retarded solutions to the wave equation by Wheeler and Feynman [81] in their absorber theory. If we assume that all radiative processes require both an emitter and an absorber, then we can say that positive mass acts as an emitter at higher temperatures and an absorber at lower temperatures. By contrast, negative mass acts as an emitter at lower temperatures and an absorber at higher temperatures<sup>¶</sup>. The bidirectional flow of entropy in systems composed of positive or negative mass also points to the photon, with its advanced and retarded solutions, as the elementary carrier of entropy in a given volume. Photons are not only the carrier of entropy in thermodynamic systems, but also serve as the elementary basis of the related quantity known as friction in the relativistic limit of mechanical systems [43, 82]. The symmetrical Second Law of Thermodynamics suggests that, in addition to absolute zero of temperature where the movement of all particles would cease, there is an absolute infinity of temperature, where the movement of all particles with a charge and/or a magnetic moment would be limited to the speed of light [82]. It may be of interest to the reader that the photon can be modeled as a composite structure composed of conjugate particles made of positive mass and negative mass [83] using the discrete mass symmetry introduced here to make the Second Law of Thermodynamics symmetrical. In this model, the photon moves as a result of the gravitational force between the negative mass particle and positive mass particle. However, the velocity of the center of momentum of the massless composite structure is not infinite but limited to the speed of light because each conjugate particle has an equal and opposite electrical charge capable of generating a magnetic field. Such a composite photon could be split by an electric field into a particle of matter made of positive mass and a particle of antimatter made of negative mass. Upon the annihilation of positive mass matter and negative mass antimatter, a massless photon or photons would be produced. Taking into consideration the thermodynamics of positive mass and negative mass discussed in this paper, the energy of a photon exchanged with a hot body would be undetermined when it was in free space, it would be deemed positive when it was exchanged with a positive mass and caused it to expand, and it would be deemed negative when it was exchanged with a negative mass and caused it to contract.

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<sup>¶</sup>Such "dark bodies," perhaps even in space, could be discovered with radiant energy sensors made of positive mass that function close to absolute zero.

We “mended” the asymmetry seen in the Second Law of Thermodynamics by introducing the coefficient  $\aleph$ , which takes into consideration the possibilities of a positive and a negative atomic mass standard. The symmetry between positive and negative mass seems reasonable and beautiful in light of the symmetrical relations between north and south magnetism [84] and positive and negative electricity [85]. Since systems composed of positive mass spontaneously evolve toward equilibrium while systems composed of negative mass spontaneously evolve away from equilibrium when exchanging radiant energy, the symmetrical Second Law of Thermodynamics suggests that the thermodynamic arrow of time [86, 87], as opposed to time itself, does not uniquely point from past to future, but points in a given direction depending on whether the system has a positive mass or a negative mass. The proposal that the thermodynamic arrow of time is symmetrical while time itself is asymmetrical is a physically realistic alternative to the quantum electrodynamical time symmetrical theory proposed by Feynman [39, 40], where time itself in our universe is symmetrical and matter proceeds forward in time while antimatter proceeds backwards in time [40]. Pitts [41] accurately stated that “*No one has ever observed such large scale time reversal or has the remotest clue of how to do it*” and proposed his own Symmetrical Time Hypothesis (STH) where the Big Bang sits at 0-time and a universe composed of matter evolves along one time axis while a universe composed of antimatter evolves along the opposite time axis. Given that neither time symmetrical hypothesis is directly testable, perhaps it is time to rethink the possibility of a discrete time symmetry when it comes to matter and antimatter and replace it with the possibility of a discrete mass symmetry between matter and antimatter [53]. Such a symmetry is consistent with the experimental and common sense experience [30, 31] that “*the great thing about time is that it goes on....an aspect of which the physicist sometimes seems to neglect.*” [88].

### Acknowledgements

I would like to thank the reviewers for their thought-provoking comments which really and irreversibly helped the evolution of this paper. As Virgil [89] wrote in Book III of the Georgics “*Sed fugit interea fugit irreparabile tempus, singula dum capti circumvectamur amore*” which means, “*But time flies meanwhile, flies irretrievable, while we, enamoured of the pleasing theme, minutely trace particulars.*”

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